ENGINEERING GRAPHICS SEMINAR

FOUR-DIMENSIONAL DESCRIPTIVE GEOMETRY

PROPOSED PROBLEMS

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ABSTRACT

There are many concepts in Four-Dimensional Synthetic Geometry that should be analyzed under the light of the Four-Dimensional Descriptive Geometry. In this paper such analysis is suggested in the form of problem proposition.

PROBLEM No. 1

Consider, in Euclidean Geometry, two planes, α and β, belonging to a line (R) (objective line or of the infinity).

A section by a third plane, γ, produces two lines (C) and (D) belonging to a point (m) of (R).

If it is assumed that the two given planes belong to distinct 3-D spaces Γ and Λ, their intersection is, generally speaking, a point (a) of the line (R), but not the line (R) itself.

It is proposed:

1. To establish the conditions to exist so that, even under the assumed circumstances (each plane belonging to distinct 3-D spaces), their intersection is a line (S) and not the point (a).

2. Verify if (S) belongs to (a) and if (S) and (R) are one and the same line.

3. Considering that plane γ belongs to a third 3-D space Ω, the intersections ($\alpha \times \gamma$) and ($\beta \times \gamma$) are two points (k) and (l), we may draw (C') parallel to (C) and (D') parallel to (D), (C') belonging to α and (D') belonging to β. Even though (C) and (D) are concurrent at a point (m) - because α and β were assumed to belong to the same 3-D space - the lines (C') and (D') are NON-CONCURRENT. However, they are not SKEW LINES, for they are non-concurrent because they belong to planes not located in the same 3-D spaces.

What figure do we obtain if, using (C') and (D') as directrices, we slide on (C') and (D') a line (X) keeping it parallel to a director plane φ?

PROBLEM No. 2

Using concepts of the projective geometry, develop the notions of parallelism and perpendicularity for the Four-Dimensional Projective Geometry. - Apply this notion to semi-parallel planes and to absolutely perpendicular planes.

PROBLEM No. 3

In the proposed problem outlined in our Special Report No. 7 - Technical Seminar Series, Engineering Graphics Seminar, Department of Graphics and Engineering Drawing, Princeton University, June 1965, if one of the given lines is parallel to a 3-D space of the system of reference for the Four-Dimensional Descriptive Geometry, the intersection of the projection of this line (the projection non-parallel to the reference line) with the reference line is the center of the involution determined by the four points where the reference line cuts the opposite sides of the complete quadrangle as shown in the mentioned paper.

1. Demonstrate that that point is indeed the center of the involution.

2. Since there are three complete quadrangles, and therefore, the possibility of three centers of involution, find a relation (analytical) among the three centers, in function of the values of the three anharmonic ratios.

PROBLEM No. 4

Develop a concept of infinity based on the solution of Problem No. 3 and compare it with that developed in Problem No. 2.

PROBLEM No. 5

Develop a study, in orthographic projection, of the hypersphere. Discuss the following topics:

a. Generation.
b. Projections of a point belonging to it.
c. Great spheres, small spheres.
d. Great circles, small circles.
e. Tangent 3-D spaces.
f. Section by a 3-D space.
g. Section by a plane.
h. Piercing points by a line.
i. Intersection of two hyperspheres.

PROBLEM No. 6

There is a group of surfaces, in three-dimensional geometry, that are susceptible of superimposition upon a plane. Develop a parallel study, in four-dimensional geometry, of hypersurfaces susceptible of superimposition upon a 3-D space. Examine the properties of the geodesic line in each of these hypersurfaces.